

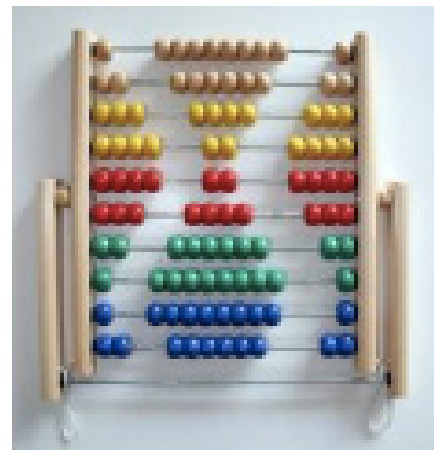
Everyone can participate in these puzzles, compare notes, and share solutions. *Enjoy!*

## Math Unplugged

Long before computers, tablets, and cell phones, people used calculators that needed no electricity or batteries. They also looked for patterns in their calculations to check their work or to be able to compute faster.

## Unplugged Calculators

You may have seen or even used an *abacus*. A version of the abacus like the one shown here is a popular toy today for young mathematicians.



Have you ever used Napier's bones to help multiply multiple-digit numbers?

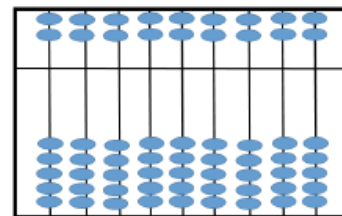
Many tools have been created throughout history to assist with numerical calculations. Find out more on your own about these mechanical calculators:

- Blaise Pascal's 17th-century calculating machine, the Pascaline
- Gottfried Leibniz's 18th-century automatic calculator, the Stepped Reckoner
- Thomas de Colar's 19th-century Arithmometer, the first machine used commercially

Has anyone you know ever used a slide rule? It may be either rectangular or circular in shape, and its scale is based on logarithms. The slide rule is used for multiplication, division, and other higher-level math calculations. At first glance, the rectangular version of the tool looks like several rulers put together. On closer inspection, you see that the center ruler moves back and forth and that the numbers get closer together on the rule as you go to the right. High school math classes in the first half of the 20th-century included use of the slide rule.

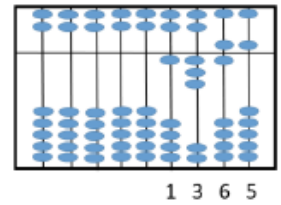
## Abacus

The abacus is the oldest known calculator on record, with historical evidence showing its use for over two thousand years. An abacus uses counting beads on parallel rods or grooves on a surface to represent numbers and to do arithmetic. The layout and grouping of the beads varied through time and location.



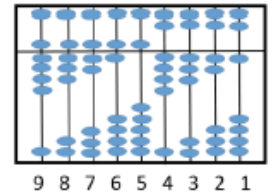
A decimal system abacus usually has a group of five beads separated by a cross bar from a group of two beads. The rods represent place values (ones, tens, hundreds, etc.). Each bead in the groups of five represent 1; the beads in the groups of two represent 5.

The first abacus shows the number 1,365. The beads representing the number are against the cross bar.



There is a 1 in the thousands place, 3 ones in the hundreds place, a 5 and a 1 in the tens place, and a 5 in the ones place.

The beads are moved back and forth to show different numbers and to do arithmetic. Adding and subtracting is done by moving additional beads to the center. Regrouping and carrying between place-value rods is done much in the same way as you do it with paper and pencil.



All of the digits from 1 to 9 are shown on the second abacus.

When the total value of beads against the crossbar gets to 10 on a rod, the amount is carried over to the next rod on the left, and the beads are pushed back to the outside.

Use your imagination to design your own abacus. One idea is to use straws and cereal with a hole in the middle. See if you can count and compute with your abacus.

## Napier's Bones

The basic multiplication facts are often shown in a table like the one shown here. In the early 17th-century, mathematician John Napier expanded this idea by separating the tens and ones digits by a diagonal line, printing the columns of numbers on circular cylinders, and creating a manual calculating machine for multiplication and division.

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

	1	2	3	4	5	6	7	8	9	0
1	0   1	0   2	0   3	0   4	0   5	0   6	0   7	0   8	0   9	0   0
2	0   2	0   4	0   6	0   8	1   0	1   2	1   4	1   6	1   8	0   0
3	0   3	0   6	0   9	1   2	1   5	1   8	2   1	2   4	2   7	0   0
4	0   4	0   8	1   2	1   6	2   0	2   4	2   8	3   2	3   6	0   0
5	0   5	1   0	1   5	2   0	2   5	3   0	3   5	4   0	4   5	0   0
6	0   6	1   2	1   8	2   4	3   0	3   6	4   2	4   8	5   4	0   0
7	0   7	1   4	2   1	2   8	3   5	4   2	4   9	5   6	6   3	0   0
8	0   8	1   6	2   4	3   2	4   0	5   8	6   6	7   4	8   2	0   0
9	0   9	1   8	2   7	3   6	4   5	5   4	6   3	7   2	8   1	0   0



## Calculation Shortcuts

Here are some shortcuts you can use to do calculations. Can you think of others? Discuss them with your friends.

### Finger Math

The multiplication facts for 9 can be revealed by looking at your hands. Hold your hands, palms down, with all 10 fingers extended. Starting from the left, if you bend the finger representing the multiple you need, you will see the product.

Show that  $2 \times 9 = 18$ . Bend the ring finger on your left hand. You have 1 finger to the left showing the 1 for the tens digit and 8 fingers to the right showing the 8 as the ones digit.

Show that  $6 \times 9 = 54$ . Bend your right thumb. You have 5 fingers to the left showing the 5 for the tens digit and 4 fingers to the right showing the 4 as the ones digit.

Practice creating the other multiplication facts for 9.

### Quick Tip

Here is a quick way to calculate a 20% tip at a restaurant. Find 10% of your total by moving the decimal point to the left one place. Then double that number.

For example, if your total bill is \$15.00: 10% is \$1.50, and 20% is \$3.00.

Practice finding tips for various amounts.

### Half the Calculations, Twice the Speed

When you need to add a group of numbers that increase or decrease by the same amount, you can pair up numbers in a way that makes the addition quick and easy.

Can you think of a way to pair these numbers for easy adding?

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$$

Try this: Add the first and last numbers. Then add the second and next-to-last numbers. What do you notice about the sums? They should be the same. Continue in this way until you get to the middle of the list. How many times did you get the same sum? Multiply the repeated sum by the number of times it occurs, and you will have the total for the entire group of numbers.

$$1 + 15 = 16, 3 + 13 = 16, 5 + 11 = 16, \text{ and } 7 + 9 = 16.$$

There are four sums.  $4 \times 16 = 64$ . The overall sum of the group of numbers is 64.

Practice finding the sums of other groups of numbers.