Family Math

math, we've got this!

Everyone can participate in these puzzles, compare notes, and share solutions. ***Enjoy!***

It will be helpful to use a quantity of small objects to create the shapes (30–40 pennies, checkers, beans, etc.). You could also grid paper or a wide tip marker on regular paper.

This Family Math activity has a separate answer key.

Geometric Numbers
Some numbers have shapes built right into them.
- In what shape are bowling pins arranged? How many pins are there?
- How many different rectangles can you form with 24 pennies?
- Why is multiplying a number by itself called "squaring?"

Geometric numbers are numbers whose quantities can be arranged in geometric shapes. Discover which numbers are geometric and how to use arithmetic to find them. Learn about the patterns in the lists of triangular, rectangular, and square numbers.

Triangular Numbers
Triangular numbers can be arranged in an equilateral triangle (the outer sides are all the same length). The first three triangular numbers are 1, 3, and 6.
Ten bowling pins are set up in a triangle so that each side of the triangle has four pins, and each row has one more pin than the row above it. The first row has one pin, the second row has two, the third row has three, and the fourth row has four.

Flip It!
If you could only move three of the bowling pins, which three would you move in order to make the triangle point in the opposite direction?

Stack It!
If you add a fifth row of bowling pins, how many pins will there be altogether? How many pins will there be in six, seven, and eight rows? Each of your answers is a triangular number. Make a list of the first ten triangular numbers. Do you notice any patterns in this group of numbers?

Predict It!
Can you think of a way to figure out the 20th triangular number without having to find the first through the nineteenth? Can you think of a shortcut for adding so many numbers?

Rectangular Numbers

Rectangular numbers can be arranged in rows and columns with the same amount (more than one) in each row or column. For the number 24, there are several rectangles that can be formed. One of them has four rows and six columns. What other rectangles can be created for the number 24? How are the rows and columns related to the number 24?

How many different rectangles can be formed for the number 36? What are they?

Why do you think there are an even number of possible rectangles for the number 24 and an odd number of rectangles for 36?

Which numbers between 10 and 20 are rectangular and which are not? Why?

Square Numbers

Square numbers can be arranged in a square with the number in each row and column being the same amount. The number nine creates a square with three rows and columns.

Squares are a special case of rectangles; square numbers are also rectangular numbers. Just as for rectangles, you can find the total number in the shape either by counting or by multiplying the number of rows times the number of columns. For squares you multiply a number by itself, so the computation is called "squaring."

What are the first ten square numbers? Of these square numbers, which can only be shown by a square, and which have other rectangles that can be formed? Why?

Coming Full Circle!

You already know that square numbers are related to rectangular numbers. Are they related to triangular numbers, too? How?
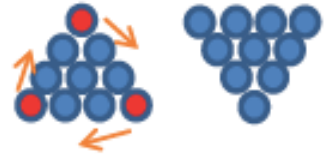
One pattern in the list of triangular numbers is that the difference between the numbers gets larger by one as you go down the list. Is there a similar pattern in the list of square numbers?

Geometric Numbers Answer Key

Triangular Numbers

Flip It!
Move the three pins on the corners (the one in the top row and the two at the ends of the bottom row).

Stack It!
The triangle with 5 rows has 15 pins; 6 rows will have 21 pins; 7 rows will have 28 pins; and 8 rows will have 36 pins.

The first ten triangular numbers are: 1, 3, 6, 10, 15, 21, 28, 36, 45, and 55.

Each new number can be found by adding the number of rows it will have to the previous triangular number.

Predict It!
The 20th triangular number is the sum of the numbers from 1 to 20.
1 + 2 + 3 + 4 + 5 ...16 + 17 + 18 + 19 + 20 + 210

Instead of adding all the numbers in order, pair the smallest number with the largest number. An amazing thing happens to make the addition quick and easy.

(1 + 20) + (2 + 19) + (3 +18) ... (8 + 13) + (9 + 12) + (10 + 11) = 21 + 21 + 21+ ... + 21 + 21 + 21

The number 21 is the sum for each of the ten pairs.
10 x 21 = 210

Rectangular Numbers
The number 24 can be arranged in 6 different rectangles with these dimensions (rows by columns): 2 by 12, 3 by 8, 4 by 6, 6 by 4, 8 by 3, and 12 by 2.

The possible number of rows and columns are the factors of 24. They show the possible ways to get 24 through multiplication. There are six different number pairs.

There are 7 possible rectangles to show 36. The dimensions for the rectangles are: 2 x 18, 3 x 12, 4 x 9, 6 x 6, 9 x 4, 12 x 3, and 18 x 2.

One of the rectangles formed for 36 is 6 by 6. Switching the numbers does not make a different shape. This rectangle is a square.

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12, 14, 15, 16, 18 are rectangular. 11, 13, 17, 19 are not rectangular. Rectangular numbers must have factors. Prime numbers are not rectangular; the only multiplication fact that equals the number is one times the number.

Square Numbers

The first ten square numbers are: 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100.

1, 4, 9, 25, and 49 can only be shown in a square. Their dimension (the length of a side) is a prime number and the only factor of the square number. 16, 36, 64, 100 can by shown in other rectangles. Their dimension is a composite number, and there are other factor pairs for the number.

Coming Full Circle!
Each square number is the sum of two triangular numbers. In fact, each square number after 1 can be created by adding two consecutive triangular numbers: 4=1+3, 9=3+6, 16=6+10, and so on.

The difference between the square numbers is always an odd number. For the first ten square numbers, the differences are: 3, 5, 7, 9, 11, 13, 15, 17, and 19. You can find the 11th square number by adding 21 to 100.

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