Family Math

math, we've got this!

Everyone can participate in these puzzles, compare notes, and share solutions. ***Enjoy!***

Fun with Infinity

This month's Family Math activity explores shapes rather than numbers. The math behind the magic you will see comes from topology, an area of geometry focused on studying and comparing shapes as they are changed, even ever so slightly. In this case, one little twist in a piece of paper leads to some surprising discoveries.

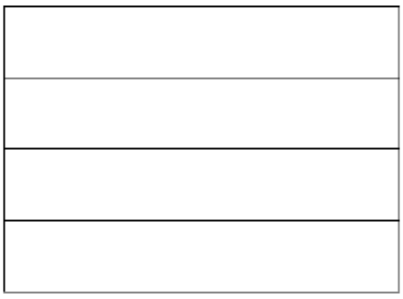
Materials

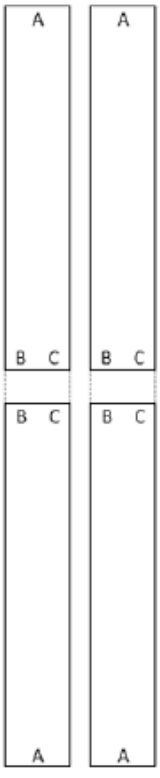
For each construction, you will need a sheet of 8½ by 11 inch paper, scissors, tape, and two different markers or pens.

The Basic Shapes

All of the constructions in this activity use strips of paper with the ends taped together. You will make two basic shapes: a loop and a loop with a twist in it, known as a Möbius strip.

- Fold a piece of paper in half lengthwise. Fold a second time to divide the lengths into four parts.





- Cut the paper on the folds to create four long and narrow strips.
- Lay the pairs of strips end to end.
- Label the top edge A in the center, the middle two edges B and C on the left and right sides, and the bottom edge A in the center as shown in the diagram.
- Tape both pairs of strips together so that the letters B and C are matched. Be sure to tape all the way across to secure the ends.
- For one of the combined strips, tape the other ends together so that the letter A is matched and visible.

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- For the other combined strip, twist one end so that the A is on the underside. Tape the ends with the letter A on opposite sides.

What a Difference a Twist Makes

Your loops should look something like the ones shown here. The letters B and C are on the outside of the back portion of the loops.

The loop that stands up straight is an example of a cylinder and is similar to the label on a can of food. For the purposes of the activities, it will be called “the cylinder.”

The loop with the twist is known as a Möbius strip because it is named for August Ferdinand Möbius, a German mathematician who was one of the first to explore the shape. Johann Benedict Listing, a mathematician exploring shapes at the same time as Möbius, is called the father of topology since he was the first to use that term and may actually have been the first to create and study the twisted loop. Möbius strips have been used in factory conveyor belts and continuous loop audio tapes because both sides of the loop get equal use. They are also seen in a popular new fashion called the infinity scarf, a Möbius strip worn around the neck.
Set your first pair of loops aside so you can compare them with the others after you have completed each activity.

Activity 1: Tracing the center path

1. Make a pair of loops (a cylinder and a Möbius strip).
2. Use one of your markers to draw a path around each loop in the center of the paper, starting at the letter A and coming back to the point where you started.
3. Describe where the path is on your cylinder.
4. Describe where the path is on your Möbius strip.
5. How are they the same? How are they different?
6. Can you tell the difference between the inside and outside of each loop?

Activity 2: Tracing a path on the side

1. Make another pair of loops (a cylinder and a Möbius strip).
2. Use a different marker to draw a path around each loop close to one edge of the paper about a third of the way from the edge of the paper either near the letter B or C.
3. Describe where the path is on your cylinder.
4. Describe where the path is on your Möbius strip.
5. How do the paths compare to each other? How do these paths compare to the ones you drew in the center?
6. Were the results what you expected?

Activity 3: Cutting the loops in half

1. You can do this activity with the loops from activity 1 with the lines in the center, or you can make another pair of loops.
2. Cut a path around each loop in the center of the paper, starting at one of the letter As and coming back to the point where you started.
3. What shapes resulted from this cut? Do you think it mattered that you cut in the center instead of on one side of the loops?

Activity 4: Cutting the loops closer to the side

1. You can do this activity with the loops from activity 2 with the lines on the side, or you can make another pair of loops.
2. Cut a path around each loop closer to one edge of the paper, starting at either the letter B or C and coming back to the point where you started.
3. What shapes resulted from this cut? How does it compare to your original loops? How does it compare to the loops with the center cut?

Fun with Infinity Answers

Activity 1: Tracing the center path

3. Answer: The path on the cylinder goes around the center of the outside of the loop.
4. Answer: The path on the Möbius strip goes around the middle of the loop, both on the inside and outside.
5. How are they the same? Answer: They both go around the center of the loop. How are they different? Answer: The cylinder's path is only on the outside. The Möbius strip's path covers both the inside and outside and is twice as long.
6. Can you tell the difference between the inside and outside of each loop? Answer: Yes for the cylinder, but not for the Möbius strip.

Activity 2: Tracing a path on the side

3. Answer: The path on the cylinder goes straight around one side of the loop.
4. Answer: The path seems to be on the left side on the outside and the right side on the inside.
5. How do the paths compare to the ones in the center? Answer: The cylinder paths are very similar; they go straight around the outside of the loop. The Möbius strip center path covers the center of the paper completely on both the inside and outside of the loop. The Möbius strip side path appears to be on the left at one point and the right at another; it does not cover both sides of the paper all the way around.
6. Were the results what you expected? Answer: Share your discoveries with each other.

Activity 3: Cutting the loops in half

3. What shapes resulted from this cut? Do you think it mattered that you cut in the center instead of on one side of the loops? Answer: You now have two cylinders of the same size. The Möbius strip is twice as long and has two twists. Predictions will vary.

Activity 4: Cutting the loops closer to the side

3. What shapes resulted from this cut? How do they compare to your original loops? How do they compare to the loops with the center cut? Answer: For the Möbius strip, you should have two twisted loops locked together. One is the same size as the original loop with one twist; the other is twice as long with two twists. The results are very different from the center cut.